

# EFFICIENCY GAINS AND STRUCTURAL REMEDIES IN MERGER CONTROL\*

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## Abstract

This paper studies the role of structural remedies in merger control in a Cournot setting where (endogenous) mergers are motivated by prospective efficiency gains and must be submitted to an Antitrust Authority (AA) which might require partial divestiture for approval. From a merger policy perspective, this paper's main contribution is two-fold. First, it shows that if mergers do not involve all firms in the industry, then merger remedies help the AA to increase consumer surplus only if assets are divested to competitors already in the market. Second, it presents a model which clarifies that there can only exist social costs to 'over-fixing' the anticompetitive effects of a merger if merger review policy treats mergers as one-time events. When a more dynamic view is taken of sequential merger review, then there can never be an 'over-fixing' problem. In this case, however, remedies are shown to be needed to make myopic merger review optimal.

*Keywords:* Merger remedies; Endogenous mergers; Efficiency gains, Merger review policy.

*JEL classification:* D43, L13; L41; L51.

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# I INTRODUCTION

When a merger review is concluded, the Antitrust Authority (henceforth AA) has a number of options: to unconditionally approve the proposed concentration, to prohibit it, or to clear it subject to commitments. In Europe, for example, the European Commission (henceforth EC) has rarely prohibited notified transactions outright. In addition, a considerable proportion of completed mergers that faced review by the EC has involved remedial conditions (the so called *merger remedies*) for particular harms.<sup>1</sup>

Merger remedies can be grouped in two different categories. (i) *Structural remedies* modify the allocation of property rights and create new firms: they include the divestiture of an entire ongoing business, or partial divestiture. (ii) *Behavioral remedies* set constraints on the merged firms' property rights: they might consist of engagements by the merging parties not to abuse of certain assets available to them, or to enter into specific contractual arrangements.

It is well known that behavioral remedies can be problematic and that the EC, in the Notice adopted on December 2000 outlining its policy in relation to merger remedies,<sup>2</sup> clearly expressed its preference for divestitures of an entire business. The Notice emphasizes that the divested assets must consist of a viable business, meaning that the business must be able to compete effectively with the merged entity. Moreover, since the viability of the divested business depends on the purchaser's suitability to run the business effectively, the EC Notice also attaches great importance to the profile of the purchaser. Hence, generally, parties involved in divestiture commitments are required to divest the relevant assets to a Commission approved purchaser that must be identified prior to the consummation of the transaction (the so called 'up-front buyer requirement').

Even though there is a wide literature on the effects of mergers on consumer and total welfare,<sup>3</sup> economic theory has not devoted much attention to the study of merger remedies.<sup>4</sup> In this paper, I study the role of structural remedies in merger control by considering a Cournot setting where mergers are motivated by prospective efficiency gains and must be submitted for approval to an AA.

To the best of my knowledge, the only paper that studies structural remedies in a Cournot framework with efficiency gains is Medvedev (2004). There exist, however, two major differences between Medvedev's framework and the setting used in this paper. First, in this paper merging parties are not exogenously given. The merger process is endogenized in the sense that firms decide on which coalition (involving all or a subset of firms in the industry) to form and include in a merger proposal. Second, while Medvedev (2004) focuses

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<sup>1</sup>For some simple summary statistics on the importance of merger remedies in the disposition of mergers by the EC, see Motta, Polo and Vasconcelos (2007).

<sup>2</sup>EC, 'Notice on remedies acceptable under Council Regulation no. 4064/89 and under Commission Regulation no. 447/98', *Official Journal*, 2 March 2001, C 68/3.

<sup>3</sup>A general discussion on the effects of mergers can be found in Motta (2004). For an economic analysis of the role of efficiency gains in determining the impact of mergers on welfare see Farrell and Shapiro (1990).

<sup>4</sup>There is, however, a strand of the literature which argues that divestitures might exacerbate pro-collusive effects of mergers if the merger-plus-divestiture industry structure turns out to be more symmetric than the status quo one (see Compte, Jenny and Rey (2002), Kühn (2004) and Vasconcelos (2005)). Another noteworthy exception is Cabral (2003) that analyzes the effects of a two-firm merger in a spatially differentiated oligopoly where the industry is assumed to be at a free-entry equilibrium both before *and* after the merger takes place, and shows that asset sales and post-merger entry are 'substitutes'.

attention on the effects of a single merger proposal, one of the main contributions of the current paper is to also deal with a dynamic merger model where a merger might trigger other mergers.<sup>5</sup>

This paper also incorporates an active AA within a merger formation game. The AA works with an enlarged tool box for merger control. In addition to blocking or unconditionally approving the merger, it can also approve the merger subject to the condition that some of the acquired assets are divested. This divestiture commitment can either give rise to the emergence of a new independent competitive entity or to the strengthening of an existing competitor not involved in the merger (merger outsider). The AA is also assumed to appraise the merger on the basis of its impact on consumer surplus. The major motivation for having a consumer-surplus-maximizing AA is that this assumption describes the current practice in the major antitrust jurisdictions.<sup>6</sup> An advantage of this assumption is that it keeps the analysis extremely simple.

Within this framework, two important merger policy implications are obtained. First, it is shown that the AA only benefits ex-post from having the remedy instrument when it can enforce a divestiture to the remaining firm in the market (unless there is a merger to monopoly), i.e. when the remedy policy enhances the efficiency of the remaining competitor outside the merger. This result, therefore, implies that, contrary to general intuition, selling capacity to a new firm (an entrant) is generally *not* the best solution for a remedy. Second, and perhaps most importantly, the proposed model clarifies that remedies can only have a negative impact on the ex-ante incentives of firms to seek for efficiency enhancing merger opportunities if merger review policy treats mergers as one-time events. As noted by Farrell (2003), the AA may have a tendency to ‘over-fixing’ the anticompetitive effects of a merger. When the AA has the possibility to conditionally approve a merger proposal, remedies are sometimes required even if the original merger (if unconditionally approved) would be consumer surplus improving. Now, if merger review policy treats mergers as one-time events, then, under some circumstances, the AA insistence on ‘over-fixing’ will cause firms not to propose some efficient mergers that: (i) would have been proposed (and approved) were divestitures not possible; and (ii) would ultimately have led to lower equilibrium prices. Nevertheless, when a more dynamic view is taken of sequential merger review as in Nocke and Whinston (2008), then it becomes clear that there can never be an ‘over-fixing’ problem. This then implies that the negative conclusion about the role of remedies suggested by the static analysis is misleading.

When a merger formation game allowing for sequential mergers is analyzed, two different scenarios are contrasted regarding the dynamic policy of an AA towards horizontal mergers. In the first scenario, the AA is forward-looking: it correctly anticipates the ultimate market structure a merger would lead to if approved. When this is the case, I show that it is always optimal for the AA to have a straight ‘up-or-down’ policy,

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<sup>5</sup>Most of the existing models on merger review treat mergers as one-time events without taking into account that a decision on a merger today affects the set of mergers that are proposed in the future. Two noteworthy exceptions are Motta and Vasconcelos (2005) and Nocke and Whinston (2008).

<sup>6</sup>As pointed out by Lyons (2002, p.1), ‘most major competition authorities operate under legislation and guidelines that reject this [total surplus] standard, and no major competition authority seems to apply it consistently. Instead, they overwhelmingly focus on consumers, including industrial consumers, to the exclusion of the welfare of merging firms.’

leaving it to the merging firm to structure the merger. The AA does not need the remedy instrument to implement its preferred market structure. In the second scenario, the AA is myopic: whenever a merger is proposed, the AA evaluates it without considering that further mergers might follow the proposed one. In this case, remedies are, however, shown to be necessary to achieve optimal decision making. In a sequential merger game with a myopic AA and a remedy option, the same outcome is achieved as that of a forward looking AA. Remedies are in this case a useful instrument since they allow the AA to evaluate mergers and remedies only on the merits of the current proposals. This result is linked to the work of Nocke and Whinston (2008) who show that, in some circumstances, an AA can implement the dynamically optimal solution by simply adopting a policy that approves mergers that do not lower consumer surplus given the current market structure. It should be highlighted, however, that while Nocke and Whinston (2008) are quite clear in their paper that myopic merger policy is only optimal in their context for disjoint merger proposals,<sup>7</sup> the current paper shows that even if this disjointness assumption is violated, the optimality of myopic merger policy can be restored when remedies are allowed.

The paper continues as follows. In Section *II*, the basic setup is described. Section *III* fully develops the equilibrium analysis and the economic intuition of the benchmark model in which: (i) attention is focused on the effects of one merger proposal; and (ii) mergers that can induce exit are assumed away. Section *IV* provides an extensive and formal analysis of the ‘over-fixing’ effect of merger remedies. Section *V* investigates the case of sequential mergers. In particular, it proposes a structure where, in each period of time, a random party is allowed to make a merger proposal. If the AA rejects the proposal, another set of parties can again make a merger proposal (but a merger proposal that was rejected once cannot be proposed again). The merger game runs until all feasible proposals are exhausted. Then firms set output. Section *VI* discusses the robustness of our main results, considering: (i) a scenario where the AA would allow firms to propose simultaneous transactions in which a firm starts by buying others and then sells a subset of the acquired assets to a firm not involved in the initial transaction; and (ii) the issue of sales mechanisms for divestiture remedies. Finally, Section *VII* offers some concluding comments.

## II THE SETUP

Consider an industry with four firms producing a homogeneous good and competing in quantities. What distinguishes firms is the amount of capital they own. The total supply of capital is assumed to be fixed to the industry (and equal to  $K$  units). Let  $k_i$  denote firm  $i$ 's capital holdings, where  $k_i \in \{1, 2, \dots, K\}$ . Hence, I normalize the smallest indivisible unit of capital assets to be one.

The cost function of a firm which owns  $k_i$  units of the industry capital and produces  $q_i > 0$  units of

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<sup>7</sup>In their setting, no firm has the possibility of being part of more than one merger.

output is given by:<sup>8</sup>

$$C(q_i, k_i) = \frac{\alpha K}{k_i} q_i + k_i f, \quad (1)$$

where  $\alpha \geq 0$ ,  $\sum_i k_i = K$  and  $f > 0$ . This cost structure was proposed by Motta and Vasconcelos (2005).<sup>9</sup> It assumes that each firm operates with a constant marginal cost of production, but the level of its marginal cost is a decreasing function of its capital holdings,  $k_i$ . In addition, it is assumed that there exists a plant specific fixed cost  $f$ , which has to be paid for each unit of the industry capital owned by the firm.<sup>10</sup> This way of modelling the cost structure aims at capturing two distinct cost effects induced by a merger. First, a merger brings the capital of merging parties into a single larger entity and, therefore, gives rise to endogenous efficiency gains. The higher the value of  $\alpha$  is, the stronger the efficiency gains induced by a merger are. Second, by creating a larger firm, a merger also has the effect of increasing fixed costs proportionally. This effect is captured by the parameter  $f$  in the cost function.

The inverse market demand is given by the twice continuously differentiable function  $P(Q)$ , where  $Q$  is the industry output. I make the following assumption on demand.<sup>11</sup>

**Assumption 1** For any  $Q > 0$  such that  $P(Q) > 0$ :

- (i)  $P'(Q) < 0$ ;
- (ii)  $P'(Q) + QP''(Q) < 0$ ;
- (iii)  $\lim_{Q \rightarrow \infty} P(Q) < \min_i \left( \frac{\alpha K}{k_i} \right)$ ;
- (iv)  $\lim_{Q \rightarrow 0} P(Q) > \min_i \left( \frac{\alpha K}{k_i} \right)$ .

Part (i) of the assumption says that demand is downward-sloping. Part (ii) is equivalent to  $\beta(Q) > -1$ , where  $\beta(Q) \equiv QP''(Q)/P'(Q)$  is defined as the degree of concavity of demand.<sup>12</sup> Parts (iii) and (iv) aim at obtaining a positive but bounded equilibrium aggregate output.<sup>13</sup> Under Assumption 1, there is a unique Cournot-Nash equilibrium. Let  $q(k_i, \mathbf{k}_{-i}; \alpha)$  and  $\Pi(k_i, \mathbf{k}_{-i}; \alpha)$  denote, respectively, firm  $i$ 's equilibrium quantity and individual profits, where  $k_i$  denotes firm  $i$ 's capital holdings and  $\mathbf{k}_{-i}$  is a vector including the capital holdings of firm  $i$ 's rival firms.

In what follows, assume that the total quantity of capital available in the industry is equal to four units ( $K = 4$ ) and that this available capital is equally distributed amongst the four firms in the status quo market

<sup>8</sup>If  $q_i = 0$ , the firm has no costs,  $C(0, k_i) = 0$ .

<sup>9</sup>This cost function is inspired by the one proposed by Perry and Porter (1985). In their framework, firms' marginal cost is linear in output and mergers reduce variable costs.

<sup>10</sup>This specification is used to rule out further scale economies due to sharing of fixed costs.

<sup>11</sup>In what follows, it will be assumed that this assumption holds for all possible market structures (and cost distributions) that may result from the proposed merger formation game.

<sup>12</sup>As is well known, Part (ii) of Assumption 1 implies that firms' best response functions are downward-sloping.

<sup>13</sup>Fixed costs should also be considered to guarantee that the aggregate output is positive (see Assumption 2 below).

structure. Assume also that there are at least two potential entrants (entrepreneurs) that have the expertise and required technology to enter in this market (at no cost) but do not have any unit of capital.<sup>14</sup>

### III THE BENCHMARK MODEL

Before Cournot competition takes place, firms play the following two-stage game with the AA.

- In the *first stage*, one firm at the status quo industry structure is randomly selected and has the opportunity to propose a merger to the AA. This firm may propose a merger with all or a subset of its rivals and will choose the merger with the highest profits (subject to the constraint of the merger policy).
- In the *second stage*, the AA decides whether to authorize or not the proposed merger. At this stage, the AA can take three different decisions: (i) unconditionally accept the proposed merger; (ii) reject the proposed merger; and (iii) accept the merger subject to the condition that some units of the merged entity capital are divested to an incumbent rival firm or to a new firm which is attracted into the market. If the AA does not authorize the merger, then the game will have come to a final node and product market competition occurs between the four symmetric firms in the status quo market structure.

The equilibrium concept is subgame perfect Nash equilibrium (henceforth SPNE) in pure strategies.

It should be remarked that if a merger is conditionally approved by the AA, then, consistent with merger enforcement in practice, the AA is assumed to be able to specify not only the number of units of capital to be divested, but also the identity of the firm to which the divested capital is sold.<sup>15</sup>

Notice also that if costs (variable or fixed) are high enough (namely, if  $\alpha$  is sufficiently high and/or  $f$  is sufficiently high), then a merger outsider is unable to make positive profits in equilibrium and will, therefore, exit the industry in case the merger goes through. The following assumption, however, guarantees that for

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<sup>14</sup>There are several industries that are characterized by fixed capacity and difficult entry. Cases in point are the cement industry (availability of raw materials and environmental regulations make new production sites unlikely) and the mineral water industry (in most countries, mineral water must be bottled at the source, and existing sources are known and already exploited). These industries are probably characterized by a low degree of efficiency gains (i.e., by a low value of  $\alpha$ ). Other industries which might fit the assumption of fixed capital are those where entry is regulated by law and subject to licenses or authorization (e.g., radio, television, telecommunication services). In many countries, the use of the spectrum for a particular purpose is given (or auctioned off) by the government. Firms can only expand by buying licenses from competitors through mergers. Very often, scale and scope economies arise when more licenses are owned by the same operator, i.e. potential efficiency gains from a merger are large ( $\alpha$  is high).

<sup>15</sup>In the U.S. Department of Justice's Policy Guide to Merger Remedies (available for download at: <http://www.usdoj.gov/atr/public/guidelines/205108.htm>), it is stated that the DoJ 'must approve any proposed purchaser' (see Section IV.D.). The same preference is also expressed in the EC Notice (paragraph 49), where it is pointed out that 'in order to ensure the effectiveness of the commitment, the sale to a proposed purchaser is subject to prior approval by the Commission. The purchaser is normally required to be a viable existing or potential competitor, independent and unconnected to the parties, possessing the financial resources, proven expertise and having the incentive to maintain and develop the divested business as an active competitive force in competition with the parties.'

all possible mergers no firm would find it optimal to exit the market.<sup>16</sup> Let  $R(q_i, q_{-i})$  denote the revenue of firm  $i$  which depends on its own output and on the aggregate output of rivals.

**Assumption 2** Let us assume that:

$$(i) \quad R(q(1, 1, 1, 1; \alpha, \mathbf{k}); q_{-i}) - \alpha(K/k_i)q(1, 1, 1, 1; \alpha, \mathbf{k}) - k_i f > 0;$$

$$(ii) \quad R(q(1, 1, 2; \alpha, \mathbf{k}); q_{-i}) - \alpha(K/k_i)q(1, 1, 2; \alpha, \mathbf{k}) - k_i f > 0;$$

$$(iii) \quad R(q(1, 3; \alpha, \mathbf{k}); q_{-i}) - \alpha(K/k_i)q(1, 3; \alpha, \mathbf{k}) - k_i f > 0.$$

Part (i) of the assumption is imposed to exclude the trivial case in which production is not viable at the status quo market structure. Parts (ii) and (iii) are about maintaining positive profits for a single unit firm in a market structure  $\{1, 1, 2\}$  and  $\{1, 3\}$ , respectively.

The following proposition identifies the consumer surplus maximizing market structures.

**Proposition 1** *There exists a unique pair  $(\alpha_1, \alpha_2)$  such that the consumer surplus maximizing market structure is  $\{1, 1, 1, 1\}$  for  $\alpha < \alpha_1$ ,  $\{2, 2\}$  for  $\alpha_1 \leq \alpha \leq \alpha_2$  and  $\{4\}$  for  $\alpha > \alpha_2$ .*

**Proof.** See the Appendix. ■

While the precise proof is given in the Appendix, I can give the main intuition for this result here. First, I show in the Appendix that, for the adopted cost function, allocating capacity  $K$  equally between all firms in the industry leads to the largest output in a Cournot model. Now, suppose that, starting from the status quo market structure  $\{1, 1, 1, 1\}$ , a two-firm merger occurs. Before the merger, the best response function for the joint production of two assets is implicitly given by the aggregation of the two first order conditions of the *separately owned assets*:

$$2P(Q_1 + Q_2) + P'(Q_1 + Q_2)Q_1 - 2\alpha K = 0, \quad (2)$$

where subscript 1 refers to the output of the two assets we are looking at and subscript 2 to the output of the others. Now, after the merger, when the two *assets are owned by the same firm*, the joint best response function is generated from the following first order condition:

$$P(Q_1 + Q_2) + P'(Q_1 + Q_2)Q_1 - \alpha K/2 = 0. \quad (3)$$

These two “first order conditions” embody the trade-off between market power and efficiencies. The merger induces a marginal cost decrease from  $2\alpha K$  to  $\alpha K/2$ , while joint “marginal revenue” is  $P(Q_1 + Q_2)$  higher

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<sup>16</sup>The motivation behind this assumption is that, in practice, exit inducing mergers (and rescue mergers) are exceedingly rare.

before the merger. Figure 1 illustrates the effects of this two-firm merger by presenting the insiders' best-response function before and after the merger.

*Place Figure 1 approximately here*

Note that there exists a critical value for  $\alpha$ ,  $\alpha_1$ , for which the (joint) best response function after the merger pivots through the original equilibrium point pre-merger but induces the same total output. Now, if  $\alpha > \alpha_1$ , then the best response function does not only pivot but also shifts outward. This will lead to higher total output and hence to higher consumer surplus.<sup>17</sup>

What happens now if a second catch-up merger takes place? At  $\alpha_1$  the merged firm resulting from the (first) two-firm merger produces the same amount as two separate firms at the status quo industry structure (i.e.,  $2q(1, 1, 1, 1; \alpha_1)$ ). Now, suppose that two separate firms face a merged firm. If  $\alpha = \alpha_1$  and these separate firms (the outsiders to the first two-firm merger) merge, their best-response to  $2q(1, 1, 1, 1; \alpha_1)$  will also be unchanged by the same argument as above. Hence, a merger from  $\{2, 1, 1\}$  to  $\{2, 2\}$  will leave output unchanged as well by the same argument as before. It has also been shown that for a higher (lower)  $\alpha$  the pivoted best response curve shifts out (in). This then implies that the second (catch-up) merger also increases output if and only if  $\alpha > \alpha_1$ . So, clearly, whenever a first two-firm merger is consumer surplus increasing, a subsequent catch up merger must be as well. This result is connected to Nocke and Whinston (2008). Focusing on a model of Cournot competition with constant returns to scale and considering potential disjoint mergers, they show that there is a form of complementarity between mergers: specifically, mergers that enhance consumer surplus continue to be consumer surplus enhancing if other mergers that enhance consumer surplus take place. In addition, in the current setting, market structure  $\{4\}$  can possibly be optimal if the cost reduction induced by a merger to monopoly is large enough.

Two more notes regarding Proposition 1 are worth making. First, the previous analysis reveals that the symmetry assumption in the model is important for the obtained result. Second, this result implies that, contrary to general intuition, selling capacity to a new firm (an entrant) is generally *not* the best solution for a remedy. The AA will only benefit ex-post from having the remedy instrument when it can enforce a divestiture to the remaining firm in the market (unless there is a merger to monopoly), in which case the remedy instrument insures that the AA can increase efficiency of the remaining competitor of the merging

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<sup>17</sup>If instead  $\alpha < \alpha_1$  there is the same pivot of the original best response function, but a shift downward. This will lead to a lower total output and hence a reduction in consumer surplus.

firms.<sup>18,19</sup> To look at divestiture formally, an additional assumption needs to be made.<sup>20</sup>

**Assumption 3** The divestiture mechanism attributes all bargaining power to the pre-approved buyer of the to-be-divested asset(s).

According to this assumption, in case a merging entity is requested to sell some asset(s) initially included in the merger project, then the pre-approved buyer of the to-be-divested asset(s) makes a take-it-or-leave-it offer to the merging firm regarding the price at which it is willing to buy the asset(s). This assumption is motivated by the fact that, in reality, very specific remedies with an approved buyer of the capacity lead to a hold-up problem, and the analyses of remedies in the US and in Europe very strongly suggest that this hold-up problem is important in practice.<sup>21</sup>

The final equilibrium outcome of the proposed two-stage game is put forward in the following proposition.

**Proposition 2** *Let  $\alpha < \alpha_2$ . Then, the final equilibrium market structures induced by the proposed merger formation game are: (i)  $\{1, 1, 1, 1\}$  (no merger) if  $\alpha < \alpha_1$ ; and (ii)  $\{2, 1, 1\}$  (two-firm merger) if instead  $\alpha \geq \alpha_1$ .*

**Proof.** This proof directly follows from the analysis of Proposition 1. If  $\alpha < \alpha_1$ , then any merger will be blocked by the AA. If instead  $\alpha_1 \leq \alpha < \alpha_2$ , then a merger proposal involving three or all firms will be remedied to generate the  $\{2, 2\}$  market structure. Furthermore, we know from the analysis of Proposition 1 that for  $\alpha \geq \alpha_1$  any two-firm merger proposal should be approved because it induces an increase in industry output. It therefore only remains to be shown that, in this region of parameter values,  $\{2, 1, 1\}$  is preferred by a merging firm to  $\{2, 2\}$  (Assumption 3 implies that in case the AA requires asset(s) to be sold by the merging firm, there are no proceeds from that sale). But this directly follows from the fact that, as shown in the proof of Proposition 1, a catch up merger (that is allowed by the AA) increases total industry output but decreases the output of the firm outside the merger. Hence, profits of the firm outside the catch up merger go down. This then implies that a merging firm prefers market structure  $\{2, 1, 1\}$  to market structure  $\{2, 2\}$ .

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<sup>18</sup>In particular, if  $\alpha_1 \leq \alpha \leq \alpha_2$  and a three-firm merger is proposed, what the remedy policy will do is to effectively engineer a merger for the firm outside the original merger proposal.

<sup>19</sup>When faced with merger proposals wherein divestiture remedies are possible, the AA uses the opportunity of the merger notifications to reshape the industry structure by reallocating the available assets in the industry so as to maximize consumer welfare. This implies that, as pointed out by Rey (2003, p. 130), there is a change in the nature of merger control since ‘introducing the possibility of remedies ... puts the merger control office in a position close to that of an industry-specific regulator’.

<sup>20</sup>In Section VI it is shown that the main results derived in the benchmark model under this assumption extend to the case where an alternative sales mechanism is at work.

<sup>21</sup>Divestiture commitments have to be implemented within a fixed time period, the length of which should in general be as short as feasible. Now, the use of up-front buyer requirement can lead to significant delay and provide substantial bargaining leverage to the designated buyer. As highlighted, for example, in the 2005 EC DG COMP *Merger Remedies Study* (available for download at: [http://ec.europa.eu/comm/competition/mergers/others/remedies\\_study.pdf](http://ec.europa.eu/comm/competition/mergers/others/remedies_study.pdf)), ‘candidate purchasers may have more scope to act strategically with delaying tactics to improve their bargaining position artificially, knowing that parties are faced with a forced-sale scenario at the end of the deadlines.’ (p.109) Along these lines, Papandropoulos and Tajana (2006) point out that the presence of an up-front buyer clause gives rise to a ‘leverage power given to the buyer, who is suddenly given a central role in the whole divestiture process’ (p.447).

The intuition behind this result can be explained by making use of Figure 2 which illustrates the impact of a catch up merger involving the two (separate) outsiders to a first merger between two firms.

*Place Figure 2 approximately here*

Let  $Q_1$  denote the output of the firm resulting from the first two-firm merger and  $Q_2$  the output of the two assets involved in the catch up merger. Since, on the one hand,  $\{1, 1, 1, 1\}$  is the consumer surplus maximizing market structure for  $\alpha < \alpha_1$  (Proposition 1) and attention is being restricted to the case in which monopoly is not optimal, on the other, the relevant region of parameter values is  $\alpha_1 \leq \alpha < \alpha_2$ . In this region, the randomly selected firm has to choose between a two-firm merger (leading to market structure  $\{2, 1, 1\}$ ) and a merger involving three or all firms which will be remedied to generate  $\{2, 2\}$  (Proposition 1). Now, as Figure 2 illustrates, whenever  $\alpha > \alpha_1$  the best response function of the firms involved in the catch up merger not only pivots through the original equilibrium point pre-merger but also shifts outward, displacing the equilibrium to a new point wherein total output increases but the output of the firm outside this catch up merger (the merged entity resulting from the first two-firm merger) *decreases*. This implies that, if a merger to monopoly is not optimal, then whenever the randomly selected firm at stage 1 proposes a merger in equilibrium, it embarks on a merger proposal for which it knows the AA cannot require restructuring through divestitures.<sup>22</sup>

## IV THE OVER-FIXING PROBLEM

When remedies are possible, the AA uses the opportunity of the merger notification to reshape the industry structure by reallocating available assets in the industry so as to maximize consumer welfare. This means it behaves so as to implement the consumer surplus maximizing market structure identified in Proposition 1. The over-fixing problem then occurs when the anticipation of remedies prevents a Pareto improving merger to be proposed.

Farrell (2003, p. 99) has pointed out that over-fixing is essentially a hold-up problem: ‘[a]n agency might be able to “hold up” the merger by demanding concessions ... [and] such demands would make the market work even more competitively ... than it would with just the merger, although (by revealed preference) the parties would not gain.’

Remedies can only be requested if a merger proposal involves three or all firms in the industry (if a two-firm merger is proposed, the AA only has a yes/no decision to make). So, over-fixing can only occur

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<sup>22</sup>Recall that each firm initially has one unit of capital in the status quo industry structure. Thus, the acquiring firm, in proposing to acquire only one of its competitors, is proposing to acquire the minimum transferable amount of capital.

when a  $\{3, 1\}$  or a  $\{4\}$  merger is proposed. Therefore, only these two cases are going to be analyzed in what follows.

**Lemma 1** *There exists a unique pair  $(\alpha_3, \alpha_4)$ , where  $\alpha_3 > \alpha_1$  and  $\alpha_3 < \alpha_4 < \alpha_2$ , such that:*

- (i) *If  $\alpha > \alpha_3$ , consumer surplus increases relative to market structure  $\{1, 1, 1, 1\}$  when a three-firm merger is unconditionally approved.*
- (ii) *If  $\alpha > \alpha_4$ , consumer surplus increases relative to market structure  $\{1, 1, 1, 1\}$  when a merger to monopoly is unconditionally approved.*

**Proof.** See the Appendix. ■

Again, the intuition can be explained making use of a simple best response function graph. Consider a proposal of a merger from  $\{1, 1, 1, 1\}$  to  $\{3, 1\}$ . Figure 3 illustrates the impact of this merger on the insiders' (joint) best response function.

*Place Figure 3 approximately here*

Let  $Q_3$  denote the output of the three assets involved in this merger and  $Q_4$  the output of the outsider firm. There exists a critical value for  $\alpha$ ,  $\alpha_3$ , for which the insiders' best response function after the merger pivots through the equilibrium point pre-merger but induces the same total output. Now, if  $\alpha$  is above  $\alpha_3$ , then the best response function after the merger does not only pivot but lies further to the right, shifting the equilibrium to a new point wherein total output (and, thus, also consumer surplus) is higher than in the status quo market structure.

Combining the results in Proposition 1 and Lemma 1 one may conclude that, when remedies can be requested, then for all  $\alpha \in [\alpha_1, \alpha_2]$  the AA will always implement the  $\{2, 2\}$  market structure even knowing that a three-firm merger and a four-firm merger (when unconditionally approved) would be consumer surplus increasing if  $\alpha > \alpha_3$  and if  $\alpha > \alpha_4$ , respectively.

The next proposition investigates what would be the final equilibrium outcome of the proposed two-stage game under a rule that any consumer surplus increasing merger must be approved by the AA. Given the analysis of Propositions 1 and 2, it is always the case that  $\{2, 2\}$  is worse for the merging firm than  $\{2, 1, 1\}$ . Therefore, a rule that every consumer welfare increasing merger must be accepted cannot create under-fixing in this model. This feature of the model then insures that the rule on consumer welfare increasing mergers is implementable.

**Proposition 3** *The final equilibrium market structures under a rule that any consumer surplus increasing merger must be approved are:*

- $\{1, 1, 1, 1\}$  (no merger) if  $\alpha < \alpha_1$ ;

- $\{2, 1, 1\}$  (*two-firm merger*) if  $\alpha_1 \leq \alpha < \alpha_3$ ;
- $\{3, 1\}$  (*three-firm merger*) if  $\alpha_3 \leq \alpha < \alpha_4$ ;
- $\{4\}$  (*four-firm merger*) if  $\alpha \geq \alpha_4$ .

**Proof.** Consider first the case in which  $\alpha < \alpha_1$ . Then, it follows directly from Proposition 1 and Lemma 1 that any merger proposal will be blocked by the AA.

Let us consider now the region of parameter values wherein  $\alpha \geq \alpha_1$ . We know from Proposition 1 that any two-firm merger must always be (unconditionally) approved in this region of parameter values since industry output goes up. Moreover, from Propositions 1 and 2, we also know that, in the very same region of parameter values, a merging firm prefers market structure  $\{2, 1, 1\}$  to market structure  $\{2, 2\}$ . This implies that under a rule that any consumer surplus increasing merger must be approved, the randomly selected firm at stage 1 will never make merger proposals that allow the AA to implement the  $\{2, 2\}$  market structure making use of remedies. Now, three different subregions should be considered. First, if  $\alpha_1 \leq \alpha < \alpha_3$ , the randomly selected firm anticipates that only a two-firm merger is unconditionally approved by the AA. Thus, a two-firm merger is proposed and the final induced market structure is  $\{2, 1, 1\}$ . Second, if  $\alpha_3 \leq \alpha < \alpha_4$ , then the randomly selected firm knows that the AA unconditionally accepts merger proposals involving two or three firms (Proposition 1 and Lemma 1). Suppose a first merger involving two firms has occurred leading to market structure  $\{2, 1, 1\}$  (we know from the previous analysis that this merger is profitable in the region of parameter values under consideration). Now, would the merged entity resulting from this first merger be willing to embark on a subsequent merger with one of the merger outsiders? More formally, is it the case that  $\Pi(3, 1; \alpha) > \Pi(2, 1, 1; \alpha) + \Pi(1, 2, 1; \alpha)$ ? The answer is positive. When a firm with two assets merges with another firm owning a single asset, aggregate profits of the merging parties must go up since aggregate fixed costs are  $3f$  both before and after the merger, but the merged entity is more efficient than any of its constituent firms before the merger. This implies that  $\{3, 1\}$  is preferred by the randomly selected firm at stage 1 to  $\{2, 1, 1\}$ . Finally, if instead  $\alpha \geq \alpha_4$ , then the randomly selected firm anticipates that a merger proposal involving two, three or all firms in the industry will always be unconditionally accepted by the AA. But, clearly, a merger to monopoly will be proposed since it maximizes industry profit. ■

An important remark regarding this result should be made at this point. In the region of parameter values wherein  $\{1, 1, 1, 1\}$  is not the consumer surplus maximizing market structure, we know from Propositions 1 and 2 that  $\{2, 1, 1\}$  is preferred by a merging firm to  $\{2, 2\}$ . This implies that, under a rule that all consumer welfare increasing mergers have to be allowed, the randomly selected firm at stage 1 will always propose mergers for which the AA decision at the following stage will have to be binary (clearance/prohibition).<sup>23</sup>

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<sup>23</sup>Put it another way, for all  $\alpha \geq \alpha_1$ , if the randomly selected firm anticipates that a merger proposal involving three or all firms would be remedied to generate  $\{2, 2\}$ , it will refrain from making this proposal since it is more profitable for this merging firm to embark on a two-firm merger (which is always accepted).

So, the availability of remedies has *no effect* in this case. The equilibrium outcome of the proposed game under a rule that all consumer welfare enhancing mergers have to be accepted then coincides with the equilibrium of the proposed game under a no remedies regime.

Now, having an AA that seeks opportunities to use its enlarged tool box available for merger control to improve market performance is not necessarily bad. The problem is that the AA insistence on over-fixing when remedies are available will, under some circumstances, induce ex-ante inefficiencies in the mergers proposed. This is shown in the next proposition.

**Proposition 4** *Let  $\alpha < \alpha_2$ . Then, there exists  $\alpha^* > \alpha_3$  such that for all  $\alpha \geq \alpha^*$ , over-fixing leads to a final equilibrium market structure wherein consumer surplus is lower than in the equilibrium market structure that would result in case merger policy consisted of a rule that any consumer surplus increasing merger must be approved.*

**Proof.** Directly follows from Propositions 1 – 3 and is omitted. ■

Hence, whenever  $\alpha \geq \alpha^*$ , the possibility of partial divestitures will cause firms to not propose large (and, hence, more efficient) mergers that: (1) they otherwise would have proposed were divestitures not possible;<sup>24</sup> and (2) would ultimately have led to lower equilibrium prices.

## V SEQUENTIAL MERGER PROPOSALS

The previous static analysis essentially shows that remedies are a bad idea since the only thing they can do is to generate an ‘over-fixing’ problem. However, in the current section, I show that a more realistic dynamic analysis reveals a *very different* role for remedies than the earlier static analysis suggested.

In what follows, I analyze a modified version of the proposed endogenous merger formation game allowing for sequential merger proposals. Consider the following game. In each period of the merger game, a random party is allowed to make a merger proposal and the AA decides to authorize or not the proposed merger. However, no party can make a proposal that has already been rejected. Let the merger game run until all feasible proposals are exhausted. Then firms set output. Two different scenarios will be analyzed: forward-looking and myopic merger policy.

Suppose first the AA is forward-looking, i.e., whenever faced with a merger proposal, the AA is able to correctly anticipate the ultimate market structure this merger would lead to (if approved). When this is the case, the following proposition shows that a straight ‘up-or-down’ policy is optimal in this setting.

**Proposition 5** *A forward-looking AA is able to implement the consumer surplus maximizing market structure with a straight ‘up-or-down’ merger policy.*

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<sup>24</sup>As Rey (2003) highlights, when remedies are possible, the AA ‘can go beyond and re-allocate the available assets in the industry, which of course has also an impact on the rates of return of those assets.’ (p. 130)

**Proof.** If  $\alpha < \alpha_1$ , one knows from Proposition 1 that the consumer surplus maximizing market structure is the status quo one,  $\{1, 1, 1, 1\}$ . So, any merger proposal will always be blocked by the forward-looking AA until all feasible proposals are exhausted. This implies that  $\{1, 1, 1, 1\}$  will be implemented.

If  $\alpha_1 \leq \alpha \leq \alpha_2$ , then  $\{2, 2\}$  is the market structure which is best from the point of view of the AA (Proposition 1). Suppose the merger game has achieved a market structure  $\{2, 2\}$ . Then, the AA would block the last remaining merger proposal (to monopoly). Now suppose the achieved market structure were  $\{2, 1, 1\}$ . Then the forward-looking AA can play the strategy of refusing any (subsequent) merger proposal that does not lead to  $\{2, 2\}$ . Since eventually only the merger proposal to  $\{2, 2\}$  is still available, the forward-looking AA knows it will accept that proposal and can wait until it is made. Suppose now we start from the status quo market structure  $\{1, 1, 1, 1\}$ . Then, by the same argument, it is an optimal strategy for the forward-looking AA to reject any initial merger proposal (involving more than two firms) that does not induce  $\{2, 1, 1\}$  until such proposal is made. Then it is accepted. Given the previous argument, the market structure  $\{2, 2\}$  will be achieved. It should be remarked that this socially desired sequence of mergers (leading to market structure  $\{2, 2\}$ ) is also privately profitable. To see that, note that profits are higher in a duopoly than in a four firm oligopoly when marginal costs are unchanged. In addition, it is straightforward to verify that, under Assumption 1, a reduction in marginal costs will increase profits for the firms.

If instead  $\alpha > \alpha_2$ , then  $\{4\}$  is the market structure which is best from the point of view of the AA (Proposition 1). This being the case, the forward-looking AA can play the strategy of refusing any merger proposal that does not include all firms in the industry. Only a merger to monopoly is accepted and the AA will wait until that proposal is made. ■

So, in such a sequential game, the forward-looking AA is always able to implement its preferred market structure with a straight ‘up-or-down’ policy and, thus, the remedy option will have *no impact* on the final outcome.

Suppose now the AA is myopic in its merger decisions, i.e., when called to decide on a given merger proposal, the AA judges it without considering that further mergers might occur. Now, contrary to what happens in the forward-looking AA case, for a myopic AA the remedy instrument is necessary to achieve optimal decision making. This is shown in the following proposition.

**Proposition 6** *The availability of remedies is necessary to make the myopic merger policy optimal.*

**Proof. (Sketch)** In what follows, let us restrict attention to the region of parameter values wherein  $\alpha_1 \leq \alpha \leq \alpha_2$ . Then, from Proposition 1, one knows that  $\{2, 2\}$  is the market structure that is best from the point of view of the AA. If the AA is myopic, only the impact of the current proposed merger on the market outcome is taken into account in its decision (the possibility of future mergers is ignored). This means that if the AA has no option for a remedy, a merger leading to  $\{2, 1, 1\}$ ,  $\{3, 1\}$  and  $\{4\}$  is (unconditionally) approved if  $\alpha \geq \alpha_1$ ,  $\alpha \geq \alpha_3$  and  $\alpha \geq \alpha_4$ , respectively, where  $\alpha_1$  is defined in Proposition 1, while  $\alpha_3$  and  $\alpha_4$  are defined

in Lemma 1. If instead it has the remedy option, it will behave so as to implement the consumer surplus maximizing market structure defined in Proposition 1. So, clearly, if a merger proposal involves three or all firms in the industry, then it will be remedied to generate  $\{2, 2\}$ . However, in the proposed sequential game, the remedy option does not have the same negative effect on the initial merger proposal as in the benchmark model. This is because even if a two-firm merger is initially approved, there will be a follow up merger to  $\{2, 2\}$  that will be myopically approved. ■

This result is closely related to Nocke and Whinston (2008). They imbed a Cournot competition framework in a dynamic model in which merger opportunities arise stochastically, and may be proposed, over time. Within this theoretical framework, they show that, in some cases, a consumer surplus oriented AA can achieve its optimal outcome by using a completely myopic merger review policy that approves mergers if and only if they do not lower consumer surplus at the time of the approval. It should be highlighted, however, that Nocke and Whinston are quite clear in their paper that myopic merger policy is only optimal in their context of disjoint merger proposals. This disjointness assumption is clearly violated in the current setting.<sup>25</sup> But still, as the previous proposition shows, the optimality of myopic merger policy can be restored when remedies are allowed. In a sequential merger game with a myopic competition authority and a remedy option the same outcome is achieved as that of a forward-looking competition authority. Again, no over-fixing problem exists. This leads to the conclusion that remedies are a useful instrument since they allow the AA to evaluate mergers and remedies only on the merits of the current proposals.

## VI DISCUSSION

### VI.1 *Alternative Merger Proposals*

A limitation of the previous analysis is that I do not allow for merger proposals in which firms approach the AA with two simultaneous transactions. First one firm buys two or three others. Then it sells a subset of the acquired assets to an incumbent rival or to (one or two) entrants which are attracted into the industry.<sup>26</sup>

As Proposition 5 shows, in a situation with sequential mergers, a forward looking AA does not need the remedy instrument to achieve optimal decision making. A straight ‘up-or-down’ merger decision rule suffices to achieve optimal decision making. Now, when simultaneous transactions are possible, this is still true since allowing for a richer set of merger proposals will simply increase the number of ‘channels’ through which the consumer surplus maximizing market structure can be implemented. Suppose that  $\alpha_1 \leq \alpha \leq \alpha_2$ . Then, as we know from Proposition 5, one possible way to implement  $\{2, 2\}$  is when the AA always rejects an initial

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<sup>25</sup>In my setting, a merger to  $\{3, 1\}$  is an alternative to a sequence of mergers to  $\{2, 1, 1\}$  and then to  $\{2, 2\}$ .

<sup>26</sup>A recent example of a transaction of this type is the *UPM Kymmene/Norske Skog/Haindl* merger case (see Kühn and Van Reenen (2008) for a detailed discussion of this case).

merger of more than two firms and allows an initial merger to  $\{2, 1, 1\}$ , which will then be followed by a catch up merger leading to  $\{2, 2\}$ . Now, one alternative way to achieve  $\{2, 2\}$  is when the AA approves two simultaneous transactions whereby the randomly selected firm buys two others and sells one of the acquired assets to the fourth firm in the status quo market structure.<sup>27</sup> The AA would then block the last remaining possible subsequent merger proposal (to monopoly).

## VI.2 *Alternative Divestiture Mechanism*

So far, I have assumed that the divestiture mechanism attributes all bargaining power to the pre-approved buyer of the to-be-divested asset(s) (Assumption 3). When remedies are requested in a three-firm merger, the to-be-divested unit of capital has to be sold to the (unique) firm outside the merger and, therefore, it seems natural to assume that this outside firm is put into an enormously strong bargaining position. However, when a merger to monopoly is proposed, then the required remedy specifies that two units of capital should be sold to an entrant. So far, it has been assumed that the AA pre-approves a specific entrant, but one may also think of a scenario in which the AA requires a divestiture to any entrant, not specifying the identity of this entrant. If this is the case, then it seems reasonable to assume instead that all potential entrants would simultaneously submit take-it-or-leave-it offers to the merging entity, specifying the price at which they would be willing to buy those two units of capital. In such a scenario, competition between the entrants would lead them to offer a payment which equals the total profit that those two units of capital would create in the merger-plus-divestiture induced market structure,  $\Pi(2, 2; \alpha)$ .

So, under this alternative divestiture mechanism, each of the shareholders in the *initial* merger proposal would end up earning  $\Pi(2, 2; \alpha)/3$  in case of a three-firm merger and  $2\Pi(2, 2; \alpha)/4 = \Pi(2, 2; \alpha)/2$  in case of a four-firm merger.<sup>28</sup> This implies that the randomly selected firm would prefer a four-firm merger to a three-firm merger. But, then, combining this information with the fact that a merging firm prefers  $\{2, 1, 1\}$  to  $\{2, 2\}$ , one finally concludes that a randomly selected firm will opt for a two-firm merger proposal for all  $\alpha \in [\alpha_1, \alpha_2]$  as in the benchmark model (see Proposition 2). Hence, the main results derived in the benchmark model under Assumption 3 extend to the case where this alternative sales mechanism is at work.

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<sup>27</sup> Another example of a possible simultaneous transaction leading to  $\{2, 2\}$  is the case where the randomly selected firm buys the remaining three firms and sells two of the acquired assets to an entrant.

<sup>28</sup> In the case of a merger to monopoly proposal, each of the four shareholders in the initial merger proposal anticipates that, if the merger goes through, it will earn  $1/4$  of the proceeds from the market of the merged entity (i.e.,  $\Pi(2, 2; \alpha)$ ) and  $1/4$  of the proceeds from sale of the divested unit of capital (which again equal  $\Pi(2, 2; \alpha)$ ). However, in the case of a merger proposal involving three firms, only proceeds from the market of the merged entity exist to be divided between the (three) shareholders.

## VII CONCLUSION

If the analysis performed by an AA shows that the effect of a proposed merger will be to substantially lessen competition in the relevant market, the AA may still decide not to block the merger. It can approve a modified version of the merger proposal where merging parties adopt certain commitments to modify the notified concentration. In Europe, for instance, the number of cases in which the EC has cleared a merger subject to remedial conditions that restructure the notified transaction has been much higher than the number of cases in which the proposed concentration was prohibited outright. Economic theory has, however, devoted very scarce attention to the study of the equilibrium impact of remedies to mergers.

The present paper studies the role of structural remedies in merger control in a Cournot setting where mergers are motivated by prospective efficiency gains and must be submitted for approval to an AA which is an active player of the game and is endowed with an enriched tool box for merger control: whenever a merger is proposed, the AA can decide to unconditionally authorize or block it, but it has also the possibility to approve the merger subject to the condition that some assets are divested to an AA approved purchaser.

Two important merger policy implications can be obtained with the proposed simple formal setting. First, if mergers do not involve all firms in the industry, then merger remedies are shown to help the AA to increase consumer surplus only if assets are divested to competitors already in the market, i.e. by creating a more efficient competitor outside the merger. Second, this paper clarifies that remedies can only generate an ‘over-fixing’ problem if merger review policy treats mergers as one-time events. When a more dynamic (and, thus, more realistic) view is taken of sequential merger review, then it becomes clear that there can never be an ‘over-fixing’ problem. In this case, however, remedies are shown to be needed to make myopic merger review optimal.

In concluding, it should be pointed out that an important limitation to the previous analysis is the fact that all firms in the status quo market structure are assumed to be symmetric ex-ante. Clearly, the assumption of ex-ante symmetry restricts the applicability of the proposed model, since real life industries are usually not symmetric in the distributions of capital among firms. So, it seems important to extend the analysis in order to consider cases where there is ex-ante asymmetry so as to try and shed light on optimal merger enforcement in situations that are not dealt with in the current model. This will be done in future research. Hopefully, the above model can be seen as a stepping stone in the direction of a more complete analysis.

## A PROOFS

Proofs of those lemmas and propositions not proved in the text follow.

## A.1 Proof of Proposition 1

The proof will be performed in three steps.

*Step 1:* In this step I will show that (for the assumed cost function) for any given number of firms in the industry  $n$ , allocating total capacity  $K$  equally between all firms leads to the largest output. Adding up the first order conditions of the  $n$  firms in the industry, one obtains:

$$P(Q) + P'(Q) \frac{Q}{n} - \alpha \sum_{i=1}^n \left( \frac{K}{nk_i} \right) = 0.$$

As is standard in Cournot models, the total equilibrium output  $Q$  is decreasing in the average marginal cost in the industry,  $(\alpha K/n) \sum_{i=1}^n (1/k_i)$ . Hence, one needs to identify the distribution of capacity that minimizes the average marginal cost in the industry:

$$\min_{k_1, \dots, k_n} \left\{ \sum_{i=1}^n \frac{1}{k_i} \right\} \text{ subject to:}$$

$$\sum_{l=1}^n k_l = K$$

Let  $F(k_1, \dots, k_{n-1}) = \sum_{i=1}^{n-1} 1/k_i + 1/(K - \sum_{h=1}^{n-1} k_h)$ . The previous optimization problem can then be rewritten as

$$\min_{k_1, \dots, k_{n-1}} \left\{ \sum_{i=1}^{n-1} \frac{1}{k_i} + \frac{1}{K - \sum_{h=1}^{n-1} k_h} \right\}.$$

The corresponding first order conditions are given by:

$$-\frac{1}{k_j^2} + \frac{1}{k_n^2} = 0 \text{ for } j = 1, \dots, n-1,$$

implying that the critical point of function  $F(k_1, \dots, k_{n-1})$  is  $\mathbf{k}^* = (K/n, \dots, K/n)$ .<sup>29</sup> So, the average marginal cost in the industry is minimized when  $k_i = K/n$  for all  $i$ .

*Step 2:* In this step I prove that there exists a unique value of  $\alpha$ ,  $\alpha_1$ , such that market structure  $\{2, 2\}$  has higher consumer surplus than any less concentrated ownership when  $\alpha > \alpha_1$ . In the status quo market structure, each firm is assumed to have one asset ( $k_i = 1$ ). Now, one can always create a best response function for the joint production of two assets. If these assets are *owned separately*, this is generated by aggregating over the two first order conditions of the separately owned assets. This yields eq. (2). If the two assets are *owned by the same firm*, the joint best response function is generated from the first order condition

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<sup>29</sup>It is straightforward to verify that the problem is quasi-convex.

(3). Now, the best response to the pre-merger output of group 2 of assets (i.e.  $2q(1, 1, 1, 1; \alpha)$ ) will be exactly  $2q(1, 1, 1, 1; \alpha)$  if and only if

$$P(4q(1, 1, 1, 1; \alpha)) = \frac{3\alpha K}{2} = 6\alpha. \quad (4)$$

One can now show that there exists a unique value for  $\alpha$ ,  $\alpha_1$ , that satisfies this equation. At  $\alpha = 0$ ,  $P > 6\alpha$ . For  $\alpha = \infty$ , we have  $P < 6\alpha$  since inverse demand is assumed to have a finite intercept. Now, it remains to be shown that, under Assumption 1, the slope  $P'4(dq(1, 1, 1, 1; \alpha)/d\alpha) < 6$ . Making use of eq. (2), one concludes that:

$$\frac{dq(1, 1, 1, 1; \alpha)}{d\alpha} = \frac{4}{5P' + 4P''q(1, 1, 1, 1; \alpha)} < 0. \quad (5)$$

The denominator in the previous expression is always negative. This is because given that  $P(\cdot)$  is strictly decreasing, then: (i) if  $P'' > 0$ ,  $\beta(Q) > -1$  implies that  $P' + P''q(1, 1, 1, 1; \alpha) < 0$ ; and (ii) if instead  $P'' < 0$ , condition  $P' + P''q(1, 1, 1, 1; \alpha) < 0$  is trivially satisfied.<sup>30</sup> Now,  $P'4(dq(1, 1, 1, 1; \alpha)/d\alpha) < 6$  if and only if  $14P' + 6P''(4q(1, 1, 1, 1; \alpha)) < 0$  which is true given that  $P(\cdot)$  is strictly decreasing and  $\beta(Q) > -1$ .

I now investigate what is the impact of a merger on the insiders' best response function. If  $\alpha = \alpha_1$ , total output remains unchanged after the merger. Only a slope effect exists. Differentiating eqs. (2) and (3), one may conclude that the slope of the best response function for the joint production of two assets before and after the merger is, respectively, given by:

$$\left. \frac{dQ_1}{dQ_2} \right|_{foc}^{separate} = -\frac{2P' + P''Q_1}{3P' + P''Q_1} < 0, \quad \left. \frac{dQ_1}{dQ_2} \right|_{foc}^{merged} = -\frac{P' + P''Q_1}{2P' + P''Q_1} < 0.$$

So, the slope effect induced by the merger is given by:

$$\left. \frac{dQ_1}{dQ_2} \right|_{foc}^{merged} - \left. \frac{dQ_1}{dQ_2} \right|_{foc}^{separate} = \frac{(P')^2}{(2P' + P''Q_1)(3P' + P''Q_1)},$$

which is always *positive* given that  $P' + P''Q_1 < 0$  by the same argument as above. So, at the critical  $\alpha_1$ , the best response function after the merger pivots through the original equilibrium pre-merger but induces the same total output (i.e.  $4q(1, 1, 1, 1; \alpha)$ ). Suppose now that  $\alpha > \alpha_1$ . The effect of any merger can be decomposed first into one where the firm post merger has  $\alpha = \alpha_1$  (giving rise to the slope effect identified above) and then  $\alpha$  is raised to the true value. A rewriting of eq. (3) gives

$$P(Q_1 + Q_2) + P'(Q_1 + Q_2)Q_1 = \alpha K/2. \quad (6)$$

In addition, raising  $\alpha$  from  $\alpha_1$  to its true value, is a move along the aggregate best response function of the

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<sup>30</sup>In case  $P(Q)$  is log-concave, then  $P' + P''q(1, 1, 1, 1; \alpha) \leq 0$  always holds along each firm's best reply (Vives (2000, p.98)).

outsiders', which is implicitly given by the aggregation of the outsiders' first order conditions:

$$P(Q_1 + Q_2) + P'(Q_1 + Q_2)Q_2 = 2\alpha K - P(Q_1 + Q_2). \quad (7)$$

Whenever  $\alpha > \alpha_1$ , the r.h.s. of eq. (6) is lower than the r.h.s. of eq. (7).<sup>31</sup> Moreover, it follows from Assumption 1 that the left-hand sides of eqs. (6) and (7) are strictly decreasing in  $Q_1$  and  $Q_2$ , respectively. So, when  $\alpha$  is raised from  $\alpha_1$  to its true value, the obtained equilibrium values  $Q_1^*(\alpha)$  and  $Q_2^*(\alpha)$  are such that  $Q_1^*(\alpha) > Q_2^*(\alpha)$ . This then implies that if  $\alpha > \alpha_1$ , the insiders' best response function does not only pivot but also shifts outward (there is also a level effect in the reaction function after the merger). This will lead to higher total output and hence to higher consumer surplus. If instead  $\alpha < \alpha_1$  there is the same pivot of the original best response function, but a shift downward, leading to a lower total output and, thus, to a reduction in consumer surplus.

At the critical  $\alpha_1$ , a merged firm produces the same amount as two separate firms (i.e.  $2q(1, 1, 1, 1; \alpha_1)$ ). Suppose now that two separate firms face a merged firm. If  $\alpha = \alpha_1$  and these separate firms merge, their best response output to  $2q(1, 1, 1, 1; \alpha_1)$  will also be unchanged by the same argument as above. Hence, a merger from  $\{2, 1, 1\}$  to  $\{2, 2\}$  will leave output unchanged as well by the same argument as before. It has been also shown that for a higher (lower)  $\alpha$ , the pivoted reaction function shifts out (in). Thus, the second (catch-up) merger also increases output if and only if  $\alpha > \alpha_1$ . (Assumption 1 is important here since the argument assumes that the best response function is a contraction.) This then completes the argument that  $\{2, 2\}$  has a higher consumer surplus than any other less concentrated market structure when  $\alpha > \alpha_1$ .

*Step 3:* In this last step of the proof, I show that there is a  $\alpha_2 > \alpha_1$  for which  $\{2, 2\}$  has higher consumer surplus than  $\{4\}$ . In market structure  $\{2, 2\}$ , there are two firms with two units of capital each. Aggregating the two first order conditions of these two separate firms, one obtains:

$$2P(Q) + P'(Q)Q - \alpha K = 0, \quad (8)$$

where  $Q$  denotes the total industry output. If the four units of capital are owned by a monopolist, then its first order condition reads:

$$P(Q) + P'(Q)Q - \frac{\alpha K}{4} = 0. \quad (9)$$

Now, the monopolist (resulting from the merger between two firms with two assets each) produces exactly

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<sup>31</sup>The right-hand sides of both equations are equal when  $\alpha = \alpha_1$  (see eq. (4)).

the same output as the pre-merger aggregate output (i.e.  $2q(2, 2; \alpha)$ ) if and only if

$$P(2q(2, 2; \alpha)) = \frac{3K\alpha}{4} = 3\alpha. \quad (10)$$

One can now demonstrate that there exists a unique value for  $\alpha$ ,  $\alpha_2$ , which satisfies this equation. At  $\alpha = 0$ ,  $P > 3\alpha$ . For  $\alpha = \infty$ , we have  $P < 3\alpha$  since inverse demand is assumed to have a finite intercept. One just needs now to show that the slope  $P'2(dq(2, 2, \alpha)/d\alpha) < 3$ . Making use of eq. (8), one may conclude that:

$$\frac{dq(2, 2; \alpha)}{d\alpha} = \frac{4}{6P' + 4P''q(2, 2; \alpha)} < 0. \quad (11)$$

Now,  $P'2(dq(2, 2; \alpha)/d\alpha) < 3$  if and only if  $10P' + 6P''(2q(2, 2; \alpha)) < 0$ , which is always true given that  $P(\cdot)$  is strictly decreasing and  $\beta(Q) > -1$  (Assumption 1).

At the critical  $\alpha_2$ , the monopolist produces the same total output the two merging parties were producing before the merger. The merger to monopoly will then only increase total output relative to the pre-merger market structure  $\{2, 2\}$  (and, hence, increase consumer surplus) if  $\alpha > \alpha_2$ . In order to conclude this proof, it remains to be shown that  $\alpha_2 > \alpha_1$ . At  $\alpha = \alpha_1$ ,  $P(2q(2, 2; \alpha)) = P(4q(1, 1, 1, 1; \alpha)) = 6\alpha$ . Now, since  $0 < 2P'(dq(2, 2; \alpha)/d\alpha) < 3$ , on the one hand, and  $P(2q(2, 2; \alpha)) < P(4q(1, 1, 1, 1; \alpha))$  for all  $\alpha > \alpha_1$ , on the other, it then trivially follows that  $\alpha_2$  strictly exceeds  $\alpha_1$ . *Q.E.D.*

## A.2 Proof of Lemma 1

I start by showing that there exists a unique  $\alpha_3 > \alpha_1$  such that market structure  $\{3, 1\}$  has higher consumer surplus than  $\{1, 1, 1, 1\}$ . Consider first the case in which the three assets are *owned separately*. The best response function for the joint production of three assets is generated by aggregating over the three first order conditions of the separately owned assets. This yields:

$$3P(Q_3 + Q_4) + P'(Q_3 + Q_4)Q_3 - 3\alpha K = 0, \quad (12)$$

where subscript 3 refers to the output of the three assets we are considering and subscript 4 to the output of the other firm. If the three assets are *owned by the same firm*, then the joint best response function is generated by the first order condition:

$$P(Q_3 + Q_4) + P'(Q_3 + Q_4)Q_3 - \frac{4\alpha K}{3} = 0. \quad (13)$$

The pre-merger output of the outsider firm (say, firm 4, owning one asset) is  $q(1, 1, 1, 1; \alpha)$ . Now, the best response by the merged entity to this output will be equal to  $3q(1, 1, 1, 1; \alpha)$  if and only if

$$P(4q(1, 1, 1, 1; \alpha)) = \frac{4K\alpha}{3} = \frac{16}{3}\alpha. \quad (14)$$

One can now show that there exists a unique value for  $\alpha$ ,  $\alpha_3$ , that satisfies this equation. At  $\alpha = 0$ ,  $P > 16\alpha/3$ . For  $\alpha = \infty$ , we have  $P < 16\alpha/3$  since inverse demand is assumed to have a finite intercept. Now, it remains to be shown that, under Assumption 1, the slope  $P'4(dq(1, 1, 1, 1; \alpha)/d\alpha) < 16/3$ . Making use of eq. (5), one concludes that  $P'4(dq(1, 1, 1, 1; \alpha)/d\alpha) < 16/3$  if and only if  $2P' + P''(4q(1, 1, 1, 1; \alpha)) < 0$ , which is always true given that  $P(\cdot)$  is strictly decreasing and  $\beta(Q) > -1$  (Assumption 1). At this critical  $\alpha_3$ , total output will be the same both before and after a three-firm merger. The three-firm merger will then increase total output (relative to the status quo industry structure  $\{1, 1, 1, 1\}$ ) if and only if  $\alpha > \alpha_3$ . Moreover, by the fact that production in  $\{2, 2\}$  is always higher than in  $\{3, 1\}$  (see Step 1 in the proof of Proposition 1), it then trivially follows that this critical  $\alpha_3$  exceeds  $\alpha_1$ .

The second part of this proof demonstrates that there exists a unique  $\alpha_4 \in (\alpha_3, \alpha_2)$  such that market structure  $\{4\}$  has higher consumer surplus than  $\{1, 1, 1, 1\}$  for all  $\alpha > \alpha_4$ . In market structure  $\{1, 1, 1, 1\}$ , each of the four assets is owned separately. Aggregating the first order conditions of all four separate firms one obtains:

$$4P(Q) + P'Q - 4\alpha K = 0, \quad (15)$$

where  $Q$  denotes the total industry output. If the four units of capital are owned by a monopolist, then its first order condition is given by eq. (9). Now, the monopolist (resulting from the merger between four firms with one asset each) produces exactly the same output as the pre-merger aggregate output (i.e.  $4q(1, 1, 1, 1; \alpha)$ ) if and only if

$$P(4q(1, 1, 1, 1; \alpha)) = \frac{5K\alpha}{4} = 5\alpha. \quad (16)$$

One can now demonstrate that there exists a unique for  $\alpha$ ,  $\alpha_4$ , which satisfies this equation. At  $\alpha = 0$ ,  $P > 5\alpha$ . For  $\alpha = \infty$ , we have  $P < 5\alpha$  since inverse demand is assumed to have a finite intercept. One just needs now to show that the slope  $P'4(dq(1, 1, 1, 1; \alpha)/d\alpha) < 5$ . Making use of eq. (5), one may conclude that  $P'4(dq(1, 1, 1, 1; \alpha)/d\alpha) < 5$  if and only if  $9P' + 5P''(4q(1, 1, 1, 1; \alpha)) < 0$  which is true given that  $P(\cdot)$  is strictly decreasing and  $\beta(Q) > -1$  (Assumption 1). To conclude this second part of the proof, it just remains to be shown that  $\alpha_3 < \alpha_4 < \alpha_2$ . First, since  $P'4(dq(1, 1, 1, 1; \alpha)/d\alpha) > 0$  (see eq. (5)), on the one hand, and  $5 < 16/3$ , on the other, it follows that the critical value  $\alpha_4$  (implicitly defined by eq. (16)) is higher than the critical value  $\alpha_3$  (implicitly defined by eq. (14)). Second, since: (i)  $P(2q(2, 2; \alpha)) < P(4q(1, 1, 1, 1; \alpha))$  for any  $\alpha > \alpha_1$ ; and (ii)  $P'2(dq(2, 2; \alpha)/d\alpha) > 0$  (see eq. (11)), it follows that the critical value  $\alpha_4$  (implicitly

defined by eq. (16)) is lower than the critical value  $\alpha_2$  (implicitly defined by eq.(10)). *Q.E.D.*

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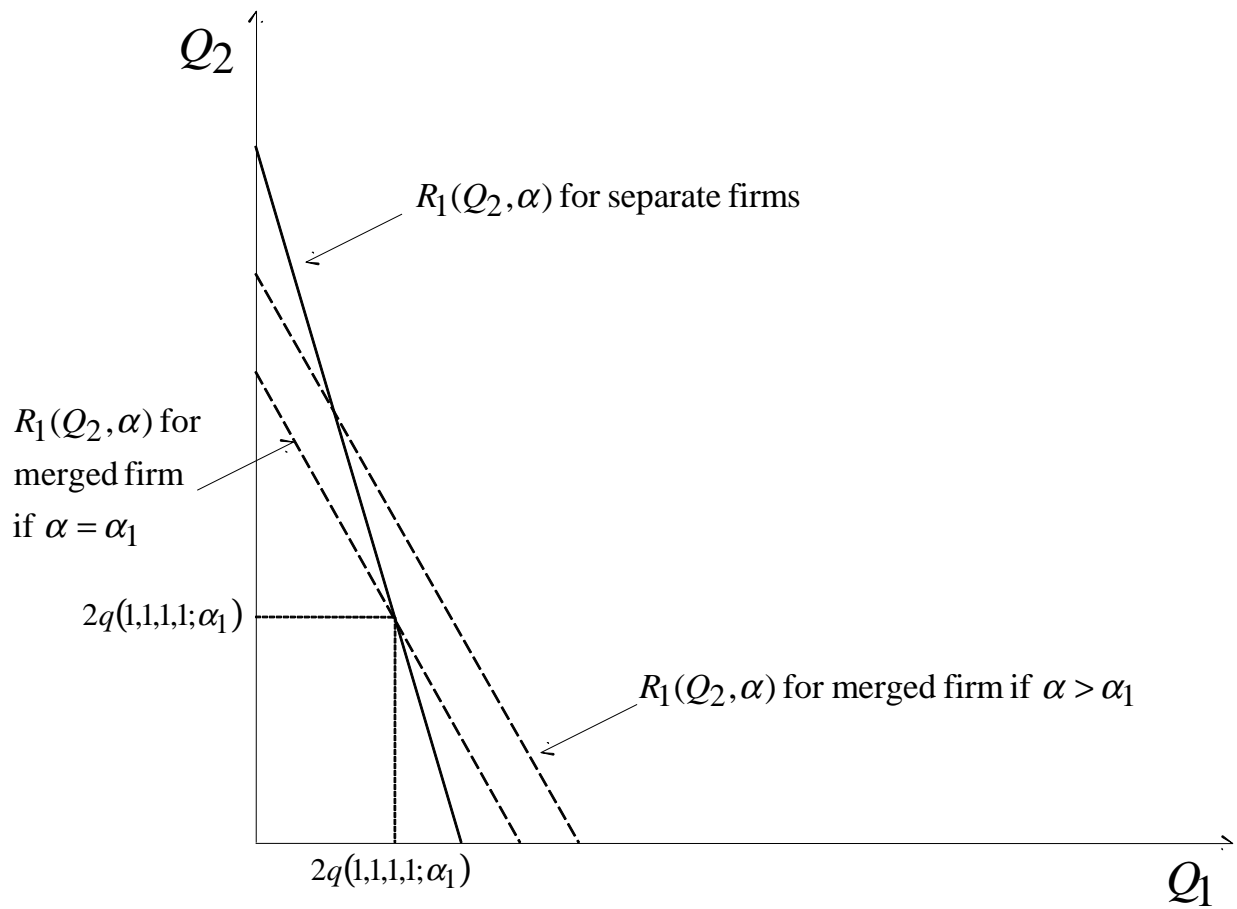


Figure 1: Two-firm Merger

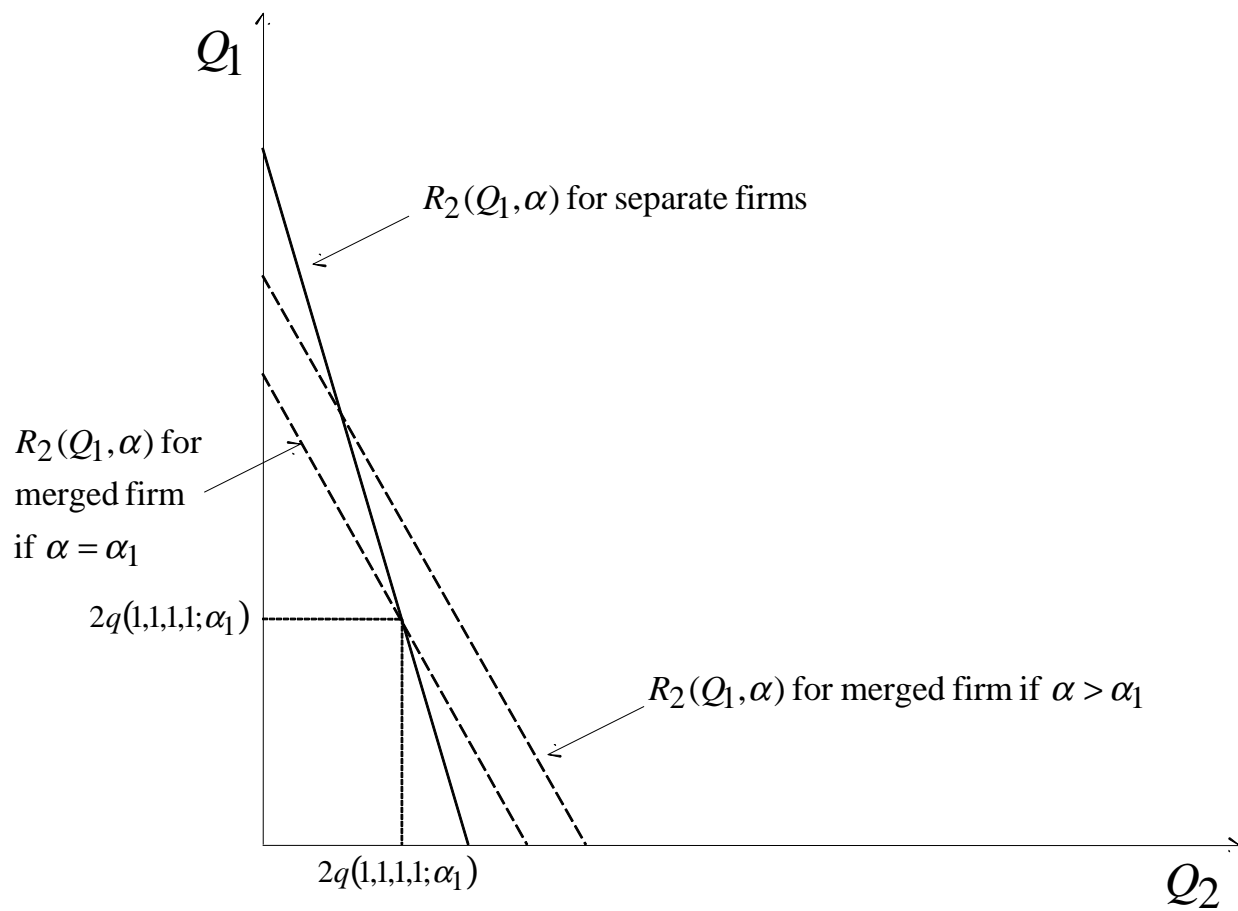


Figure 2: Catch up Merger

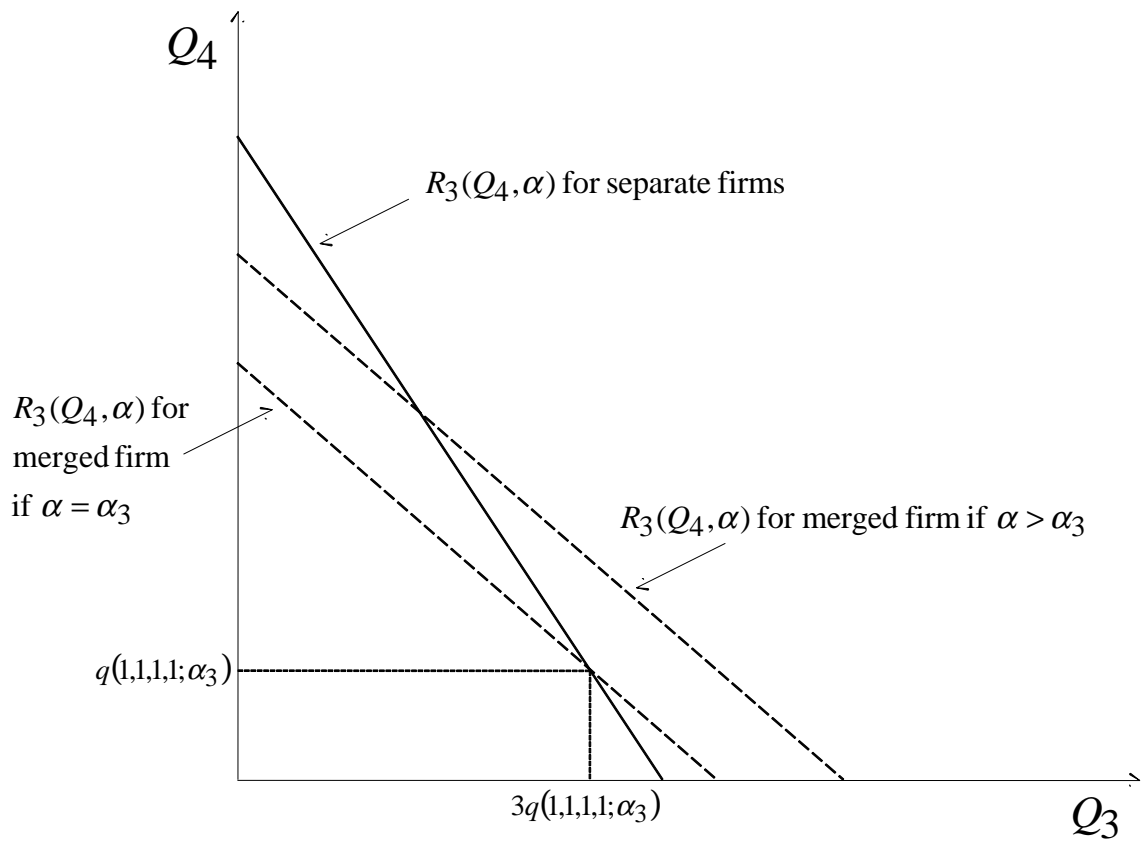


Figure 3: Three-firm Merger